Implicit Differentiation

So far, most functions have been expressed in **explicit form**. Some, however, are defined **implicitly**.

For example:

Implicit Form	Explicit Form	Derivative
$x^2y=1$		

Examples:

It is not always possible, however, to solve for *y* explicitly. For example, $x^2-2y^3+4y=2$. In these cases, we must use **implicit differentiation**.

The key to finding $\frac{dy}{dx}$ implicitly is understanding that the differentiation is happening *with respect to x*.

- When you differentiate terms involving *x* alone, you can differentiate as usual.
- When you differentiate terms involving *y*, you must apply the Chain Rule.

Ex. 1: Differentiate each of the following:

a) $\frac{d}{dx}\left[x^3\right]$

b) $\frac{d}{dx}\left[y^3\right]$

c) $\frac{d}{dx}\left[x+3y\right]$

d) $\frac{d}{dx}\left[xy^2\right]$

<u>Guidelines for Implicit Differentiation in equations</u>

1. Differentiate both sides of the equation *with respect to x*.
2. Collect all terms involving $\frac{dy}{dx}$ on the left side of the equation and move all other terms to the right side of the equation.
3. Factor $\frac{dy}{dx}$ out of the left side of the equation.
4. Solve for $\frac{dy}{dx}$ by dividing both sides of the equation by the left-hand factor that does not contain $\frac{dy}{dx}$.

<u>Ex. 2:</u> Find $\frac{dy}{dx}$ given that $y^3 + y^2 - 5y - x^2 = -4$.

<u>Ex. 3:</u> Find all points of horizontal and vertical tangencies for the graph of $x^2 + y^2 = 1$.

Ex. 4: Determine the slope of the tangent line to the graph of $y^3 - xy = -6$ at the point (7, 2).

Ex. 5: Determine the slope of the ***normal*** line of $x^2 + 4y^2 = 4$ at the point $(\sqrt{2}, -\frac{\sqrt{2}}{2})$.

Ex. 6: The position function of particle moving along the y axis is given by
t = siny where t is in seconds and y is in ft.
Find the velocity function (dy/dt).

Find the acceleration function.

What is the velocity at t=π?

Is the particle speeding up or slowing down at t=3π/4?

Try:

Find $\frac{dw}{dt}$: $2\sin w\cos t = \pi^2$

Second derivatives…ugh

Ex. 7: Given $x^2 + y^2 = 25$, find $\frac{d^2y}{dx^2}$.

Ex. 8: Given $x^2 + xy = 5$, find $\frac{d^2y}{dx^2}$.